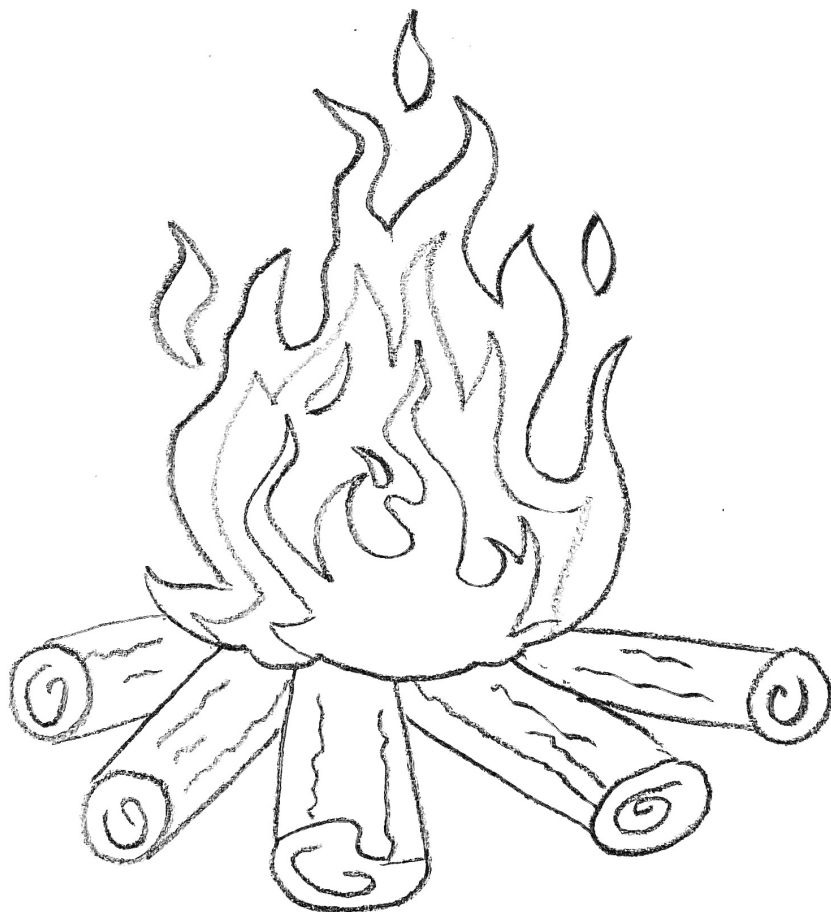


KOREŠPONDENČNÝ MATEMATICKÝ SEMINÁR

MATIK

Číslo 3 – Ročník 36

matik.strom.sk



Ahoj!

TC z- ÿ-Y ç-sbe\$ MATIK->Vbq eq\$— f\bcpf qC C^S @q-PCUs qGi a Vq\
 zbPb> CUC ebsY@^ f zb\ zb s\ Gszp> UC f ^S bç^ -Uz \ > Cs ^ \ eq\$P-@-U
 -Uebx f^W%eqC z <P ^-USMf^CU <P < f-si y s- \ ~ zCS ^- b@ C^~ fb Hq\ C
 z @ bf Pb ^Czq @s^ Pb s szqC@^S f b4WbeC^ sWYCY\ S ç-sz^ WS- fC@ <S S
 , Ws- zSz- \ zC^zbq < ^Ceb@ qB @bsz- >^G HÜ dCf^CfCq\ G C^ -4-@ <Cs- szCQ
 4b~ ~fS@\ CF

fC@ <SMATIK-



Vzorové riešenia 2. série úloh zimného semestra



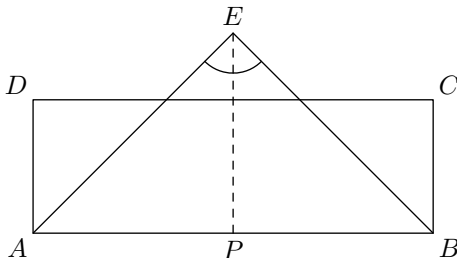
beq fbf- $\mathbb{S} = \mathbb{X} \mathbb{C}^{\wedge} \mathbb{W} \mathbb{O} - \mathbb{V} \mathbb{C} - \mathbb{X} - \langle \mathbb{W} \mathbb{V} \mathbb{Y} \mathbb{C} \mathbb{b} \mathbb{f} -$
 $\wedge - \mathbb{U} \mathbb{q} \mathbb{U} \mathbb{S} \mathbb{C} \mathbb{q} \mathbb{C}^{\wedge} \mathbb{S} = \mathbb{T}^{\wedge} \mathbb{W} \} \mathbb{q} \mathbb{4}^{\wedge} \mathbb{b} \mathbb{f} - \mathbb{p} \mathbb{S} \mathbb{b} \mathbb{d} \mathbb{q} \mathbb{W} \mathbb{C} \mathbb{q}$

vI $\mathbb{q} \mathbb{C}^{\wedge}$

Zadanie

a $P^{\wedge} S W z f q b 4 @ ^ S W A B C D \setminus - s z q ^ \sim A B @ P \{ s z b e \% r s z q ^ \sim B C @ P$
 $c s z b e \sim i 3 b @ E \mathbb{U} \mathbb{C} \mathbb{f}^{\wedge} \mathbb{S} \mathbb{Y}^{\wedge} e b \mathbb{Y} s z b e \% b @ s \mathbb{C} \mathbb{W} \% C D > \mathbb{Y} \setminus \setminus b b 4 @ ^ S W A B C D$
 $- j C E j = j D E j, W \mathbb{U} \mathbb{C} \mathbb{f} \mathbb{C} \mathbb{W} s \sim P \mathbb{Y} A E B m$

Riešenie



, $\mathbb{S} \setminus \mathbb{C} j C E j = j D E j > \mathbb{C} \mathbb{S} \mathbb{C} 4 b @ E \mathbb{Y}^{\wedge} - b s s \mathbb{C} \mathbb{C} \mathbb{W} \% C D i \mathbb{S} z b P b \mathbb{U} \langle \mathbb{U} \mathbb{f}^{\wedge} > C E$
 $\langle - \mathbb{q} \mathbb{f} \mathbb{C} \mathbb{Y} - \mathbb{U}^{\wedge} - b s S A B > \mathbb{V} \mathbb{C} \mathbb{Y} C A B C D \mathbb{U} \mathbb{C} b 4 @ ^ S W a s \mathbb{C} \mathbb{C} \mathbb{W} \% \mathbb{U} \mathbb{C}^{\wedge} - @^{\wedge} s \mathbb{C} \mathbb{C} \mathbb{W}$
 $\mathbb{W} \mathbb{Y} - @ \mathbb{C} \mathbb{Y} \mathbb{U}^{\wedge} - e b \mathbb{Y} \mathbb{f} \mathbb{S} \mathbb{G}, \mathbb{W} z \mathbb{C} @ s e \sim z \setminus \mathbb{C} \mathbb{W} \mathbb{Y} \mathbb{S} \sim \langle 4 b @ E^{\wedge} - A B > \mathbb{U} \mathbb{U} e^{\circ} z$
 $P s - 4 - @ \mathbb{C}^{\wedge} - \langle P @ - \sim e q s z q @ A B i a @ \mathbb{S} f^{\wedge} \mathbb{Y} f - > C @ \mathbb{W} s \mathbb{C} \mathbb{C} \mathbb{S} \mathbb{W} A P - U P B \mathbb{U} \mathbb{C}$
 $e b \mathbb{Y} \mathbb{f} \mathbb{S} - A B > \mathbb{C} \mathbb{S} \mathbb{C} 1,5 s z b e \% a \mathbb{S}^{\wedge} \mathbb{S} \mathbb{C} s \mathbb{S} - \mathbb{C} \mathbb{f}^{\wedge} \mathbb{S} \mathbb{Y}^{\wedge} b s E b @ P \mathbb{U} z \mathbb{S} \mathbb{C} 1,5 f \mathbb{W} \mathbb{q} z \mathbb{S}$
 $s z q^{\wedge} - b 4 @ ^ S W - e b \mathbb{Y} s z b e \% g y - \mathbb{W} \mathbb{C} z \mathbb{q} \mathbb{U} \mathbb{P} b \mathbb{Y} \mathbb{W} \% A P E > B P E s \mathbb{C} \mathbb{f}^{\wedge} b \mathbb{q} \setminus \mathbb{C}^{\wedge}$
 $- e q \mathbb{f} b - P \mathbb{Y} s b \langle - \mathbb{W} @ - \setminus S A E - B E i \mathbb{S} f \% \delta @ \mathbb{f} - \sim P \mathbb{Y} \% o f z \langle P z z \mathbb{q} \mathbb{U} \mathbb{P} b \mathbb{Y} \mathbb{W} \mathbb{C} P$
 $4 - @ z @ \mathbb{C} \mathbb{f}^{\wedge} - \mathbb{W} \mathbb{f} \mathbb{C} \mathbb{W} > - z b 45 f s \mathbb{C} z f \mathbb{C} z \mathbb{W} \langle P \sim P \mathbb{Y} \mathbb{f} f z \mathbb{q} \mathbb{U} \mathbb{P} b \mathbb{Y} \mathbb{W} \mathbb{U} \mathbb{C} 180 \mathbb{g}$
 $, \mathbb{V} \mathbb{C} \mathbb{Y} \mathbb{C} - P b \mathbb{Y} A E B s - s \mathbb{W} @ - \langle \mathbb{f} b \langle P z - \mathbb{W} \langle P z - P \mathbb{Y} \mathbb{f} > \mathbb{U} \mathbb{P} b \mathbb{f} \mathbb{C} \mathbb{W} s \mathbb{U} \mathbb{C} 90 i$

Komentár

a $4 @ q - \mathbb{Y} s \setminus \mathbb{C}^{\wedge} - b x - \mathbb{U} \mathbb{f} \mathbb{C} - s \mathbb{W} \mathbb{C} \mathbb{Y} \langle P \mathbb{q} \mathbb{C}^{\wedge} i, \mathbb{U} z > \mathbb{C} b^{\wedge} \mathbb{U} \mathbb{W} z \mathbb{S} \mathbb{C} 4 b @ \% e b s z q \leftarrow \mathbb{Y} \setminus f^{\circ} \mathbb{C} -$
 $z \mathbb{q} \mathbb{C} \mathbb{S} \mathbb{Y}^{\wedge} z \setminus > \mathbb{C} s \mathbb{f} b \mathbb{U} \mathbb{C} \mathbb{q} \mathbb{C}^{\wedge} \mathbb{S} @ b s z - z \mathbb{C}^{\wedge} \mathbb{C}^{\wedge} \langle b @ \mathbb{f} b @ \mathbb{S} \mathbb{S} \mathbb{R} \mathbb{W} \mathbb{Y} \mathbb{S} \mathbb{C} b \mathbb{L} b \setminus \mathbb{C} z \mathbb{q} \mathbb{f} \mathbb{W}$
 $\mathbb{Y} P \sim >^{\wedge} - e q 4 \mathbb{G} e b @ \mathbb{P} 4^{\wedge} P b e b e \mathbb{S} - \mathbb{W} \mathbb{f} \% \mathbb{q} \mathbb{f} \mathbb{C} \mathbb{Y}^{\wedge} \mathbb{S} - \mathbb{q} \mathbb{C}^{\wedge} \mathbb{S} \mathbb{U} @^{\wedge} b @ - \langle P b^{\wedge} \mathbb{G} z - \mathbb{C} i d b \mathbb{q}$
 $s \mathbb{S} z \mathbb{q} 4 - @ \mathbb{f} - \mathbb{U}^{\wedge} - \mathbb{q} \mathbb{a} \mathbb{b} \mathbb{f} -^{\wedge} \mathbb{S} i r e q \mathbb{f} \mathbb{S} s \mathbb{S} e \mathbb{q} \mathbb{S}^{\wedge} b 4 \mathbb{q} - b \mathbb{W} s \mathbb{S} - \mathbb{C} \mathbb{U} \mathbb{C} @ b 4 \mathbb{q} > - 4 \%$
 $s \setminus \mathbb{C} \mathbb{Y} \mathbb{P} \mathbb{C} \mathbb{Y} e \mathbb{S} e b \mathbb{C} \mathbb{p} \sim \setminus \mathbb{C} \mathbb{S} -^{\wedge} \mathbb{S} \mathbb{C} b s \mathbb{W} s \mathbb{S} \mathbb{S} f^{\circ} \mathbb{C} b \mathbb{C} \mathbb{p} \mathbb{f} - > - \mathbb{Y} \mathbb{C} \mathbb{q} \mathbb{P} b @^{\wedge} \mathbb{C} z b^{\wedge} \mathbb{C} \setminus \mathbb{C} \mathbb{C}$
 $e b \mathbb{f} - \mathbb{b} \mathbb{f} - \leftarrow @ \mathbb{W} i d \mathbb{q} \mathbb{S}^{\wedge} b s \mathbb{q} \mathbb{a} \mathbb{b} \mathbb{f} -^{\wedge} \mathbb{S} \mathbb{U} z b \mathbb{S} f @ \% b 4 \mathbb{C} @ \mathbb{C}^{\wedge} + \mathbb{g}$



beq fbf Y-[- qS „ Vbe% Vbec-^%o ~4b , - qlbfç W JDçC C
^- Wq USC çC C S=O- ^W Bq@ Ybf- pSb dçWç

Zadanie

} | G ^- ^- çC^S^bfCbsYfCs <PYe<S- @çfç-z- W @ sW @ \ s- 4-ÿ ebx^→
- Y4b ^Geb^ -fCzb f^-U^ ^ g , S\ C C^- bsYfCUCc| <PYe<bf- W @ < ^SP
ebx^-eqfCv @çfç-zi ySC fS\ C CW @ @çfç- ebx^-çf^-W ebçCz <PYe<bf
- W bsz-z^ @çfç-z+Vb W UC^- bsYfC@çfç-zm| -UçCf CZWδ b ^bszS- @bW-zC
CS ^CS i

Riešenie

Šb<- @ ^S fS\ C C^- bsYfCUCc| <PYe<bf- W @ <PYeC ebx^-eqfCv @çfç-zi
Š zbPb f%Y f > CUC^- zCzb bsYfCu| q <^%P <^→ bszi y- WSC fS\ C CW @
@çfç- ebx^-çf^-W ebçCz <PYe<bf i , 4%zbzb eYZSb\ \ ~s 4% ebçCz <^→ bsz
@çC^ ebçzb\ @çfç-zi Š zbPb f%Y f > CebçCz @çfç-z \ ~s 4% @çC b\ çsY
u| ? çC çsY u| s c> |> {> J> v> D> _> c|> cD | J> {v - u| ? çfç-z \ ~s 4%
\ S S - YCv>z WCS P \ C 4%v>D> _> c|> cD | J> {v - u|
[C Cs ebxçC >- W 4%zb f%P-@- Y=

dbçCz @çfç-z	dbçCz <^→ bsz @çfç- -
v	c
D	-
-	D
c	v
cD	J
J	{
{v	
u	c

[C^USP 4% ^C bPY>eqCz CUC^ <PYeC ebx^-v @çfç-z- fS <zC ^SC>eqCz C
^- UfS < \ C 4%zb W @çfç-z- W W UC <^→ bszi , W4% C4b Y çsY u| @çC^
ebçzb\ @çfç-z-z- W4% ^→ bszS 4% ^C bPY 4% çf^-b\ Cq^C çx Y C \ C @ S @çfç
ç-z+

Komentár

r \ Cq @S CszC \ ^bP <fY@S z b YP~ f%ebçz- ^- eY ebçCz 4bçbf i , C- < f-s
f- W^C^- Y f CZW%çC C^S i dçzb f @%b@ebq ç- \ C eq\ %SC sS eqçb 4%
^C bPY C Szb f- ÿ- YS \ b ^bs i , %B^CzS- z WsS- <S C^C^- UCzC f CZWδ b Q
^bszS- z WSC sS- <S C s <C SP ^- UCzC f CZW% Y 4- @C C^ zC f→ szP^
ç-s 4bçbf <- ^C@W^ç^SC YP%TC @4q sSeqzb ^- U s%sz \ f P- @ ^ \ b Q
^bsz - \ b ^bsS z Wf S ^CzC C \ - U ^SCb sebYç^ >- ~W-CzC Cz zb sebYç^
fYsz^bs \ ~sS \ - f CZW%seqf^C \ b ^bszS

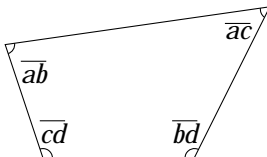


beq fb f - a s W q ; - < q - d C b V b f < s
 ^ - W q U S C q C C S = , Y 3 - S z b f - | S P - Y , b a q W

{ D q C C }

Zadanie

V b ~ S - < \ - \ ~ z \ - z f - q z f b q - P b Y W i , C W s z S ~ P b f z f b q - P b Y W s q s Y
 < Y C ^ < < S C q a > b > c > d - W ^ - b 4 q < W i] - U S C f C z W % a b ^ f C W s z S ~ P b f - f % Q
 s f C z S C e q C b S f C W s z S ~ P b f ^ C f % P b f - U i a 4 q < b W C S 4 - S Y - s z q q ^ i



Rieszenie

d q C C C ^ s z q C 4 ^ - U e q f ~ f C @ b \ S > C W G y C W b ~ S - \ - z f - q z f b q - P b Y W > s q C z
 U P b f ^ z b q ^ < P ~ P b f U C q f ^ 360 i S z b P z e q C @ b W @ s S f S C C < b s z - f S q f ^ S ~ =

$$\overline{ab} + \overline{ac} + \overline{cd} + \overline{bd} = 360$$

y b > C s - q s Y - ^ - P - @ - ^ - \ S z C @ C s z b W < ^ - \ C ^ - > C s - z - z b < S t t ^ - < P - @ -
 f q s Y C C W i z z W C s z z b q f ^ S ~ f S C C ~ e q f S @ b ^ - S Y @ b f ^ P b z f - q i

$$10a + b + 10a + c + 10c + d + 10b + d = 360$$

$$20a + 11b + 11c + 2d = 360$$

V G y C q s Y b > c > d s < S t t z W s ^ - < P - @ - U f q x s P ~ C E _ i , W s s - f C z W % ^ S P
 @ b s - @ \ C ^ - U j ^ q S - P b @ b z - > _ > < S z \ C > C P b @ b z - 11b + 11c + 2d \ C @ b s - P b f -
 P b @ b z - ^ - U j S < | c v i] - U S S - \ b ^ P b @ b z - a s S f S C C @ b q - z - W b (360 216) :
 20 = 7 ; 2 i O b @ b z - q s Y C a \ ~ s 4 % < C Y - > z @ a 4 - @ C 4 - y D - Y 4 b _ i p b x b 4 C q \ C
 b 4 - e q e - @ %
 , W a = 8 >

$$20 \cdot 8 + 11b + 11c + 2d = 360;$$

$$11(b + c) = 2(100 - d);$$

- f - s z q ^ - q f ^ S C U C \ U f ^ C @ C S C ^ - U C C ^ - s z S S - z C @ z b S z \ ~ s e Y z S - U e q C
 e q f s z q ^ - i V G y C | ^ S C U C @ C S C ^ U C C ^ - s z S S - 100 d \ ~ s 4 % @ C S C ^ U C C
 @ C ^ - s z S S y b e Y z Y C ^ f e q e - @ C C d = 1 > W y C d U C < S t t f q x s P ~ b @ C e b _ i
 ? b s - @ \ C d i

$$11(b + c) = 2(100 - 1);$$

$$b + c = 18:$$

$20 \cdot 9 + 11b + 11c + 2d = 360;$
 $11(b + c) = 2(90 - d);$
 $11(b + c) = 2(90 - 2)$
 $b + c = 16$

$$20 \cdot 9 + 11b + 11c + 2d = 360;$$

$$11(b + c) = 2(90 - d);$$

$11(b + c) = 2(90 - 2)$
 $b + c = 16$

$$11(b + c) = 2(90 - 2)$$

$$b + c = 16$$

$y \in \{ [b; c] \mid C \in [9; 7] \times [8; 8] \times [7; 9] \}$
 $CzW \setminus b \wedge bszSs =$

$CzW \setminus b \wedge bszSs =$

$$\acute{a} \ a = 8 > b = 9 > c = 9 > d = 1 \ f89 + 89 + 91 + 91 = 360g$$

$$\acute{a} \ a = 9 > b = 9 > c = 7 > d = 2 \ f99 + 97 + 72 + 92 = 360g$$

$$\acute{a} \ a = 9 > b = 8 > c = 8 > d = 2 \ f98 + 98 + 82 + 82 = 360g$$

$$\acute{a} \ a = 9 > b = 7 > c = 9 > d = 2 \ f97 + 99 + 92 + 72 = 360g$$

Komentár

, $\circ \ S^- \ C \ S \ C \ b \ f \ ^- \ Y \ f \ C \ W \ J \ C \ C \ S \ Y \ P \] \ S \ W \ b \ q \ C \ S \ C \ S \ f \ - \ W \ k \ f \ b \ Y \ S \ e \ q \ C \ Q$
 $s \ W \ - \ ^ \ S \ \ b \ ^ \ b \ s \ z \ - \ ^ \ C \ @ \ b \ s \ z \ - \ z \ b \ C \ @ \ d \ L \ - \ C \ ^ \ z \ f \ - \ Y \ S \ C \ f \ % \ W \ - \ Y \ ^ \ - \ b \ x \ - \ U \ f \ C \ z \ W \ \ b \ Q$
 $\ ^ \ b \ s \ z \ S \ \check{S} \ - \ z \ - \ W \ z \ b \ e \ b \ s \ z \ - \ e \ s \ \ C \ \ - \ s \ C \ S \ z \ q \ ^ \ 4 \ b \ @ \ % \ , \ C \ q \ \ C \ \ C \ f \ 4 \ - \ @ \ < \ ^ \ b \ s \ z \ 4 \ - \ @ \ C \ z$
 $e \ C \ z \ - \ W \ < \ P \ z \ b \ C \ C \ S \ < \ P \ b \ e \ - \ z \ q \ ^ \ C \ U \ > \ - \ 4 \ % \ s \ \ C \ f \ \ \ \ b \ P \ Y \ - \ @ \ C \ S \ e \ Y \ e \ b \ C \ z \ 4 \ b \ @ \ b \ f \ i \ = \ g$



$beq \ f \ b \ f \ - \ Y \ = \ [\ - \ q \ S \ [\ - \ s \ q \ ^ \ - \ - \ [\ - \ q \ S \ , \ R \ S \ V \ q \ " \ ? \ \sim \ @ \ U \ W$

$\ ^ \ - \ W \ q \ U \ S \ C \ C \ S \ = \ [\ S \ P \ - \ Y \ , \ b \ @ \ C \ W$

$I \ C \ C \ C \ C$

Zadanie

$y \in \{ C \in S \mid \ ^ \ @ \ C \ p \ \ s \ \ - \ \ - \ \ - \ Y \ ^ \ - \ @ \ b \ \ \leftarrow \ b \ \ \ Y \ P \ b \ \ \leftarrow \ \ \ - \ z \ C \ \ - \ z \ S \ W \ % \ 0 \ \ - \ @ \ \ Y \ f \ C \ z \ W \ % \ 0$
 $e \ q \ f \ b \ C \ s \ Y \ > \ W \ b \ q \ s \ \ ^ \ C \ @ \ \ Y \ \ - \ e \ s \ \ - \ W \ s \ \ C \ z \ @ \ f \ b \ < \ P \ \ \ Y \ C \ \ < \ P \ \ C \ s \ C \ Y \ d \ b \ \ \ z \ C \ S \ \ S \ P$
 $\ ^ \ - \ \check{S} \ \ - \ @ \ b \ W \ - \ z \ C \ \ C \ S \ @ \ ^ \ C \ y \ - \ Y \ S \ ^ \ C \ C \ \check{S} \ z \ - \ U \ i$

Riešenie

$V \ - \ @ \ e \ q \ f \ b \ C \ s \ Y \ b \ W \ C \ \ | \ s \ \ ^ \ C \ e \ - \ q \ ^ \ C \ - \ z \ @ \ P \ b \ f \ S \ \ C \ \ - \ e \ s \ \ S \ 4 \ \ - \ W \ s \ \ C \ z \ ^ \ C \ e \ - \ q \ ^ \ C \ P \ b$
 $\ - \ e \ - \ q \ ^ \ C \ P \ b \ C \ s \ Y \ i \ \ | \ \ - \ U \ C \ \ S \ ^ \ C \ e \ - \ q \ ^ \ C \ \ Y \ C \ \ \ C \ s \ Y \ \ C \ \ - \ ^ \ - \ U \ C \ \ S \ C \ e \ - \ q \ ^ \ C \ \ Y \ C \ \ \ C \ s \ Y \ \$
 $\ C \ J \ i \ d \ q \ z \ b \ \ ^ \ - \ U \ C \ \ S \ e \ q \ f \ b \ C \ s \ Y \ > \ W \ b \ q \ f \ S \ \ C \ \ - \ e \ s \ \ - \ W \ s \ \ C \ z \ @ \ f \ b \ < \ P \ \ \ Y \ C \ \ < \ P \ \ C \ s \ C \ Y$
 $\ C \ \ C \ i \ y \ C \ q \ \ \sim \ W \ \ \ \ C \ \ - \ U \ W \ \ @ \ \ ^ \ C \ e \ - \ q \ ^ \ C \ C \ s \ Y \ \ f \ % \ C \ \ - \ W \ \ c \ \ f \ S \ \ C \ z \ \ W \ b \ \ \leftarrow \ e \ s \ \ i$

V- @ e-ēCçsYb f°ç SC -Wb | fSC C<-es- -Wb 2k>V@C k > 1>zC@ UC Y C^ i
y-WC ~4bfb ^ ^Ge-ēCçsYb-seb c{ \ C C<-es- -Wb 9+2k eqC^UJW k > 1i
y-Wb <-es- ^CfSC C f CZW%eqfbçsY \ C^ SC -Wb c{>zb UGz |>}>I>u- cci

Komentár

req-f^CqC^C^Sebxbsz-f Yb < @fb<P ç-szi dqfb- 4bYb @bW- > CçsY |>}>I>u>cc
s- sWzbç^C^C@ U <-es- -Wb s çCz @fb<P eqfbç sCYf4-ÿ -q-~\ C^zb\ -Wb fb f^bQ
qfb\ qC C^ >- Y4b f°es-^ \ f CZW<P \ b ^bsz g ? q-Pb- ç-s b- 4bYb @bW- >
C f CZW%bsz-z^ eqfbçsY s ~ z-Wb <-es- @ Ui 3bP- S >\ ^bP < f-s U@~
< z <Pzb ç-sz eYC f°G-P- Ysçb fSC@b ^Cf°B^~z^CWszq zC 4b@bfi dçeb@b4^ <P
YbP-P zqC4 \ %SC ^- zb> C@ W< UC W\ eYZ^ >- Wçs @bW- ^ b4Cç-sS
zfqC^S i



beq fb f- Y- [- qz^ \ SY -W [- qz^ zCfWb
^ - Uq USC qC C^S=, Y 3 -Y^zbf-

|J qC C^

Zadanie

] CbsS< fbY^UJWçsYb ki | -sY^CWUsW%UbszC^CeçsSYy -Y<- ^-e s- Y^UJW-P
k + 2 q <^%P W^@ <P <CY <P çsCY ? bW-zC> C \ C@SçsY \ S^ -szC^s- ~qS^C
^ -<P-@- 4-ÿ @fbUS- çsCYs çx @Sb\ @CS^ \ 2k>- Y4b @fbUS- çsCYsb s çzb\
@CS^ \ 2ki

Riešenie

dbx qS C- ^- <f°dW%eb @CC^ 2ki dbsWeS^WU CsSSP z-W- 4% f°dW%z- 2k z
4bYb f U@^CUsWeS^WU Š f°dW% - k sfbUe-q^C - U fY4b 0 \ -e-q2k>çS C0>- k
\ -e-qkg rWe ^ ebzb\ \ → C(2k - 2) : 2 + 2 = k + 1i
æsCY- zC@ <f°dW%f> \ → C k + 2i db@- ? SPSYçzbfPb eqS<e~ z-WçSsz-UC-sQ
eb U@- sWeS^W>@b WbqUe-zçf -seb @fCçsY ^-es-^ ^- sz^G, Ws SP
<f°dW%eb @CC^ 2k qf^-W>çsY b@çz \ C- çx @SY4~@C@CS^ 2ki, Ws q<^C>
< @C^ <CsWeS^SW^ → çsY sz-ç sçz - @bsz ^C Cs çCz @CS^ 2ki

Komentár

, Wb \ Cz fS@C > qC C^SC 4bYb ^-bx-UU@^b@-P >-W^ -PY szC @bsz Y seq-f^%
^ -e- @ [q ^-s zC@ > C seq-f^%P qC C^ f- qC C^ f 4C-g 4bYb Y^ \ -Yi ; P<CS
4% s\ Ceçzb ~ebx bq^S ^- ^S Wb W^ Geçf \ %P eb\ <bW
? q-P - zçS fCz- f<-@ ^ \ C C<bf Cb4C^S ^- zb> CP-@ \ C ^UJW sWQ
eS^W çsCYf sWeS^CçsCYs fC \ S eG-S <W\ ebçzb\ eqWf f k + 2g yC^zb f^bq
f°ç S^b ^-ebfC@ → Ce U@Cb Sz -eYW<S ? SPSYçzbfPb eqS<e~i db@- ebsY@Q
^ <P e-qsbf> V@C Pbfq \ Cb @CS^ ^bsz^G^ → b- f2kg b WbqU^C → C S @C
S^Hq -<S^ ^-fS <\ C Cs fCW- eq f@Ceb@b4^bs b- ebfC@ > Cs- 4-@C \ ~Q
sC ebx Cq ^- <f°dW% d b- sz \ Sz @fb\ S^Hq -<S \ S^ → sz-ç ~ Y^ @C ^bf-
seq-f^CsWeS^W% f°dW%f - qC C^S>- Wb UC zb f^bqf > \ → Cs Wb ç<- @ q b i



beq fbf- S-[S₁W Obqf-zPbf- [SP-YI - sq'-
^- W₁ USC qC C'S =, Y 3-S'zbf-] S'- O-@W₁f-

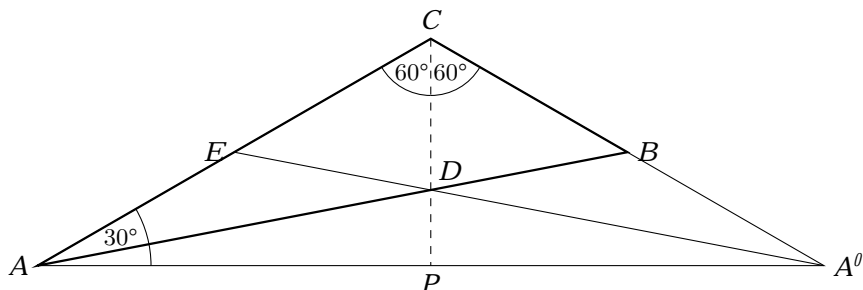
|J qC C'

Zadanie

Odpz, ^@CqfPbb zCe- lCzf- q- zcpUPbY' W ABC>W@C@ W sCqW%AC lCc| e-Ybf
- fCwS ~PY ACB lC 120 sz-e bfi a s zbPzb ~PY eqZ ^- szq ^- AB f 4b@C Di
y- WSC fS\ C> CjAEj = jECj = jCBj>W@CE lCszq@szq ^%ACi Vb W e-Ybf \ -
sCqW CDM? W lCebzqC4^ f%W@f eqS^C- zC@ 4G eb\ b-Sq\sbj- ^Si

Riešenie

CD lCbsb~ ~PY ACB>z WCPb @CY ^- ebY%} PY%ACD - DCB 4-@ \ - zC@ b4-
eb 120 /2 = 60 i ySC fS\ C> CjCBj = jAEj = jECj = jACj/2 = 12/2 = 6i
? bWCS\ C sS sCqW EDi , S ^S C sS> C zcpUPbY W%EDC - BDC s <Pb^
eb@- fCZ%&-s fszq ^~ DC \ - U sebYc^ >~PY%DCE - DCB \ - U eb 60 - szq ^%o
EC - CB s <b<-@ ^S qf^~ W @P g r @bW^< bsbfb s \ Cq^ eb@- bsSCDi



p- YUs<bszq\ C 4b@ A^> Wbq 4-@C bsbfb s \ Cq^ s 4b@b\ A eb@- bsSCDi
, y- W s \ Cq^bszS 4b@bf A - A^ eb@- CD lC CD WYX ^- AA^i Rq eqCSq^ W
sS^<fS\ C Pi , ^SW~z zcpUPbY WAA^C \ ~s 4%oqf^bq \ C^ ^>Y4b sCqW AC
lCbsbfb s \ Cq^s A^C>- zC@ s qf^~ W @P i y- WC CP lC f W^ ^- <~W@ ~
AA^ qf^bq \ C^ ^ Pb zcpUPbY W AA^C f lC zb WYX S- ^- <~W@ ~ eqCP-@- U<
eqzS- PY \ fq-PbY\ g

, qf^bq \ C^ ^b\ zcpUPbY W f W> - ^S- - bs~PY ^- <~W@ ~ seYf- U f lC^~
sCqW> qSC PC lC - U - ^Sb- zbPzb zcpUPbY Wi VGy C jCAj = jCA^j = 12
- jCBj = 6>B lCszq@szq ^%CA^ - AB lCeb@b4^C - ^Sb- f zcpUPbY W AA^Ci
3b@D lCzCq< - S W\ zcpUPbY W AA^C>eqzb CY ^- eqCSq^ W @fb-P - ^<i
, <@S Y^bs CD s 2/3 f.@S Y^bsSCP>eqzb C - S W @CY - ^S- eqfCf eb\ Gq
2 : 1i, W Sz \ C@ W CP>4-@C C fC@C @bq-z @ W DCi

, zcpUPbY W AA^C eb^>C ~PbYbeczS <~W@^S j<ACA^j = 120 i } PY%eqS
<~W@^Ss qf^~ W fCW>z WC sSSP \ C C@bq-z <G s Cqz f^ zbq^ <P ~PYbf
f zb\ zb zcpUPbY W - W (180 120)/2 = 30 i dbx qC C- 4S SC ^- zcpUPbY W

APCi yC^zb \ —~P%db fC WszS <P 90 > 60 - 30 i yC^zb zpbU-PbY WLC fYsz^C
ebYfSb~ qf^bszq ^^ Pb zpbU-PbY W ACC^>Wbq 4%gf^SMYbsbf \ <b4q<C^ \
4b@- C <G bs APi TCPbszq ^- CC^0 4%gf z-W\ eqe-@C4bY qf^- Wb@P— Wb szq ^-
AC c|i CP UCebYfSb~ szq ^%CC^0 - UC@P—vi
r zq ^- AP \ —v>- zC@ CD 4~@C@P—6 (2/3) = 4i

Komentár

y-zb YP- 4bY eb\ Cq^C ^-epç^→çb s- S b@qW@SB ^- 4b@bfb\ Pb@^bzC^ i
dqCb 4%6\ C f\ <P<CS ebsWz^ <be-q@b4q <P q-@@b 4~@ <^-i g
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Lb^S\ Czçz- WCsU ebWUCeb CzçC^ ^GSwq TCzC @ YC Sz seb\ C^ Szb>
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^G- bWq Pbf- f sXCbW^ - @G- zS^ çsY q@CU^G-P- UC f sXCbWf zf- qC^CQ
UWPb f q<~>^b ^G- bWq PYC^ i, @b zçzS-C UC f @%fC\ S@ YC Sz be s- W @
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ebzCSs pbf^ Wb ^-s ebzCSb - Uzb> CszCs- ^C4-Sb@bf<@ - UçS szbç^ qC C^S >
eqCb CSVçy ^C4bY eY C@bzS P^~z @b Wb^<~>z Ws- s YPb~ ebe- sbf- Ysz- zbç^C
- f%4bWf- YszCSs<- ^C^C -Y ebzC4^ <P 4b@Wbfi g

Konečné poradie zimného semestra 36. ročníka

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	, Y^W 3~S^zbf—	Š_	3K Or~ç	IJ	-	-	-	-	-	-	cĚ
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	, Y^~; PY@^—	Šu	K, [æ, 3,	IJ	-	-	v	-	-	-	cĚ
Ii	O- ^- Bq@Wbf—	Šu	K, [æ, 3,	IJ	-	-	D	-	-	Q	cĚ
vi Qui	yb\ —; - 4 W	Šu	ŠOYKX	I	-	-	-	J	{	u	—
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c{i QcJi	~4b Csz-W	ŠD	Š, fb3,	Ju	-	-	v	-	-		Q
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cvi	, bžb 3~S^z	Šu	; ŠpŠ-Š,	Ju	u	-	-	{	Q	Q	DJ
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ci	OCšEOçš	ŠD	Ksfy, VB	I	-	J	-	u	D	-	uI
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I CE QI ci	X- ~q dCf- ~Wbf-	Šu	ŠVčJVB	{ I Q Q CE Q Q Q D
	[SP- Y, ~CW	ŠD	ŠVčJVB	cv u Q I Q Q Q Q D
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